

**Compressible Flow Subroutine Library  
Reference Manual (C/C++)  
DD-00008-010E**

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# 1 About this Guide

## 1.1 Legal Information

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## 1.3 Introduction

This manual describes the *Application Programming Interface* (API) of the *Compressible Flow Subroutine Library* for the C and C++ programming language families.

## 1.4 Audience for This Guide

The audience of this guide is assumed to be C or C++ programmers who understand the basic concepts of at least one of the aforementioned programming languages.

Familiarity with pointer based arrays in C/C++ is strongly recommended.

## 1.5 How to Use This Guide

This guide first describes some general programming details of the library and then documents each function individually.

## 1.6 Conventions Used in This Guide

*x*

Normal math typesetting represents a normal variable.

**x**

Bold math typesetting represents a vector.

Mono

Monospace typesetting represents C function names, variables or data types.

## 2 Overview

### 2.1 Performance Characteristics

Each function or subroutine has a declared performance characteristic, these represent how the performance of a function can be predicted.

**Fixed** The function always executes around the same amount of instructions. Depending upon the actual values passed in the arguments there might be insignificant variations but do not contain any iterative solver. The behavior of standard library functions used is not considered for this performance classification.

**Iterative** The function uses an iterative root finding algorithm to compute the result. In the library all functions which use iterative solvers have a maximum amount of iterations, after which they will fail.

Iterative solvers are usually non-linear root finding methods such as Newton-Rapshon, False Position or Bisection style solvers or can be solvers for differential equations such as Runge-Kutta methods as described in [DKahaner1988].

### 2.2 Error Behavior

All library subroutines where an error may occur usually have an optional integer pointer argument. If the latter is given and an error occurs the error code will be written into the integer at the pointer's location. If no error occurs the integer will not be changed. As such this allows by design a logical-OR behavior in which one may call several subroutines and check at the very end whether an error code has been set.

This error code is also set into a *thread-local* variable and can be inquired using the 3.1.3 function for the current thread. The latter will also provide the name of the offending function for easier traceback.

Functions that are designed for specific purposes may however return NaN (*Not-a-Number*) or infinity  $\infty$ . The specific error behavior for those is documented for each of such functions individually.

## **3 Function Reference**

### **3.1 System functions**

#### **3.1.1 Introduction**

The functions in this group provide system information about the CFSL library and error handling facilities.

**3.1.2 arsyver - Get version information**

```
void arsyver(short *major, short *minor, const char **cmplr);
```

Return the major, minor version and the compiler used to compile the library.

**Arguments**

**MAJOR - SHORT INTEGER** *EXIT: (optional)* Pointer to store major version in.

**MINOR - SHORT INTEGER** *EXIT: (optional)* Pointer to store minor version in.

**CMPLR - CHAR POINTER** *EXIT: (optional)* Pointer to store a pointer to the compiler version string.



**3.1.3 arsyerr - Retrieve and reset error code**

```
int arsyerr(const char **fn, bool clear);
```

Returns the last error code and messages that occurred in the current thread.

**Parameters**

**FN - CHAR POINTER** *EXIT*: (optional) Pointer to store a pointer to the function name string.

**CLEAR - BOOLEAN** *ENTRY*: Whether to clear the error code from the *thread-local* context.

## 3.2 Characteristic Mach Number

### 3.2.1 Introduction

This function group computes the characteristic Mach Number  $M^*$  as given in [JDAnderson1982] and [Shapiro1953] from a given mach number  $M$  or in reverse the latter given  $M^*$ . The characteristic mach number has the following properties:

$$\begin{aligned} M^* &= 0 && \text{if } M = 0 \\ M^* &= 1 && \text{if } M = 1 \\ M^* &< 1 && \text{if } M < 1 \\ M^* &> 1 && \text{if } M > 1 \end{aligned}$$

And furthermore the its most useful property:

$$\lim_{M \rightarrow \infty} M^* = \sqrt{\frac{\gamma + 1}{\gamma - 1}}$$

The following properties are computed:

Table 1: Characteristic mach number properties

<b>I</b>	<b>Symbol</b>	<b>Description</b>
1	$M$	Mach number
2	$M^*$	Characteristic mach number

**3.2.2 armcmm - Compute given normal mach number,  $M$** 

```
double armcmm(double g, double m, int *ierr)
```

Compute given normal mach number,  $M$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M > 0$ .

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status Codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M \leq 0$ .

**Return Value**

Characteristic mach number  $M^*$  or NAN on error.

**3.2.3 armcms - Compute given characteristic mach number,  $M^*$** 

```
double armcms(double g, double mstar, int *ierr)
```

Compute given the characteristic mach number,  $M^*$ . Note that if  $M^* = \sqrt{(\gamma+1)(\gamma-1)}$  the function will return  $M = \infty = \text{INFINITY}$ .

**Performance**

Fixed

**Parameters**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**MSTAR - REAL ENTRY:** Characteristic mach number  $0 < M^* \leq \sqrt{(\gamma+1)(\gamma-1)}$ .

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status Codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-3** Given  $M^*$  out of specified range.

**Return Value**

Normal mach number  $M$  or NAN on error.

### 3.3 Hugoniot compression

#### 3.3.1 Introduction

Hugoniot's equation can be used to model a normal shock wave by thermodynamic constants only as given by [JAnderson1982]. As such this equation allows the relation of the pressure factor over the shock wave  $p_2/p_1$  to the density factor over the same  $\rho_2/\rho_1$  given a specific heat constant  $\gamma$  without being related to a velocity or mach number. Note that the Hugoniot based shock wave compression does not yield exactly the same results as normal shock wave relations and in fact deviates from them at higher mach numbers. The following properties are computed:

Table 2: Hugoniot compression properties

<b>I</b>	<b>Symbol</b>	<b>Description</b>
1	$p_2/p_1$	Pressure factor
2	$\rho_2/\rho_1$	Density factor

**3.3.2 arshud - Compute given density factor  $\rho_2/\rho_1$** 

```
double arshud(double g, double d, int *ierr);
```

Compute pressure factor based upon density factor  $\rho_2/\rho_1$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

D - **REAL ENTRY**: Density fraction  $0 < \rho_2/\rho_1 < \sqrt{\frac{\gamma+1}{\gamma-2}}$ .

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Given  $\rho_2/\rho_1$  out of range.

**Return Value**

Pressure fraction  $p_2/p_1$  or NAN on error.

**3.3.3 arshup - Compute given density factor  $p_2/p_1$** 

```
double arshup(double g, double p, int *ierr);
```

Compute pressure factor based upon pressure factor  $p_2/p_1$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

P - **REAL ENTRY**: Pressure fraction  $p_2/p_1 > 0$ .

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

-1 Specific heat ratio  $\gamma \leq 1$ .

-2 Given  $p_2/p_1$  out of range.

**Return Value**

Density fraction  $\rho_2/\rho_1$  or NAN on error.

### 3.4 Critical mach number



**3.4.1 armcrm - Critical pressure coefficient given  $M_{\text{crit}}$** 

```
double armcrm(double g, double mcrnt, int *ierr);
```

Compute given the critical pressure coefficient given  $M_{\text{crit}}$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**MCRIT - REAL ENTRY:** Critical Mach number  $0 < M_{\text{crit}} < 1$ .

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M_{\text{crit}}$  out of range.

**Return Value**

Critical pressure coefficient  $C_p$  or NAN on error.

**3.4.2 armcrc - Critical mach number given  $C_p$** 

```
double armcrc(double g, double cp, int *ierr);
```

Compute given the critical mach number given  $C_p$ .

**Performance**

Iterative

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

CP - **REAL ENTRY**: Critical pressure coefficient  $C_p$ .

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Given  $C_p$  out of range.
- 3 Iterative solver failed.

**Return Value**

Critical mach number  $M_{crit}$  or NAN on error.

**3.4.3 armcrpg - Critical mach number given  $C_{p0}$  using Prandtl-Glauert pressure approximation**

```
double armcrpg(double g, double cp0, int *ierr);
```

Compute critical mach number given  $C_{p0}$  Prandtl-Glauert pressure approximation.

**Performance**

Iterative

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

CP0 - **REAL ENTRY**: Critical pressure coefficient  $C_{p0}$ .

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

-1 Specific heat ratio  $\gamma \leq 1$ .

-2 Iterative solver failed.

**Return Value**

Critical mach number  $M_{\text{crit}}$  or NAN on error.

**3.4.4 armcrla - Critical mach number given  $C_{p0}$  using Laitone pressure approximation**

```
double armcrla(double g, double cp0, int *ierr);
```

Compute critical mach number given  $C_{p0}$  Laitone pressure approximation.

**Performance**

Iterative

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

CP0 - **REAL ENTRY**: Critical pressure coefficient  $C_{p0}$ .

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

-1 Specific heat ratio  $\gamma \leq 1$ .

-2 Iterative solver failed.

**Return Value**

Critical mach number  $M_{\text{crit}}$  or NAN on error.

**3.4.5 armcrkt - Critical mach number given  $C_{p0}$  using Karman-Tsien pressure approximation**

```
double armcrkt(double g, double cp0, int *ierr);
```

Compute critical mach number given  $C_{p0}$  Karman-Tsien pressure approximation.

**Performance**

Iterative

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

CP0 - **REAL ENTRY**: Critical pressure coefficient  $C_{p0}$ .

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

-1 Specific heat ratio  $\gamma \leq 1$ .

-2 Iterative solver failed.

**Return Value**

Critical mach number  $M_{\text{crit}}$  or NAN on error.

## 3.5 Isentropic flow

### 3.5.1 Introduction

This function group computes the thermodynamic properties of isentropic flow for a calorically perfect gas as described in [naca1135] with one-dimensional variable changes. Isentropic flow assumes the following continuity equation:

$$\rho AV = \text{const}$$

The momentum equation is:

$$p + \rho AV^2 = \text{const}$$

The following properties are computed:

Table 3: Isentropic flow properties

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M$	Mach number
2	$p_0/p_1$	Pressure ratio
3	$\rho_0/\rho_1$	Density ratio
4	$T_0/T_1$	Temperature ratio
5	$A/A^*$	Critical area ratio

**3.5.2 arfism - Isentropic relations given  $M$** 

```
void arfism(double g, double m, double result[5], int *ierr);
```

Compute the isentropic relations given the mach number  $M$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M > 0$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 3.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.

**3.5.3 arfisp - Isentropic relations given  $p_2/p_1$** 

```
void arfisp(double g, double p, double result[5], int *ierr);
```

Compute the isentropic relations given the pressure ratio  $p_2/p_1$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

P - **REAL ENTRY**: Pressure ratio  $0 \leq p_2/p_1 \leq 1$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 3.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $p_2/p_1$  out of range.



**3.5.4 arfisd - Isentropic relations given  $\rho_2/\rho_1$** 

```
void arfisd(double g, double r, double result[5], int *ierr);
```

Compute the isentropic relations given the density ratio  $\rho_2/\rho_1$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

R - **REAL ENTRY**: Density ratio  $0 \leq \rho_2/\rho_1 \leq 1$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 3.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $\rho_2/\rho_1$  out of range.

**3.5.5 arfist - Isentropic relations given  $T_2/T_1$** 

```
void arfist(double g, double t, double result[5], int *ierr);
```

Compute the isentropic relations given the temperature ratio  $T_2/T_1$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**T - REAL ENTRY:** Temperature ratio  $0 \leq T_2/T_1 \leq 1$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 3.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $T_2/T_1$  out of range.

**3.5.6 arfisa - Isentropic relations given  $A/A^*$** 

```
void arfisa(double g, double a, bool is_sub, double result[5], int *ierr);
```

Compute the isentropic relations given the critical area ratio  $A/A^*$ .

**Performance**

Iterative

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**A - REAL ENTRY:** Critical area ratio  $A/A^* \geq 1$ .

**IS\_SUB - BOOLEAN ENTRY:** Whether the mach number corresponding to  $A/A^*$  is assumed to be sub- or super- sonic.

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 3.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $A/A^*$  out of range.
- 4 Iterative solver failed

## 3.6 Fanno flow

### 3.6.1 Introduction

This function group computes the thermodynamic properties of adiabatic flow with friction for calorically perfect gas as described in [JAnderson1982].

The following properties are computed:

Table 4: Fanno flow properties

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M$	Mach number
2	$p/p^*$	Pressure ratio
3	$\rho/\rho^*$	Density ratio
4	$T/T^*$	Temperature ratio
5	$V/V^*$	Velocity ratio
6	$p_{01}/p_{02}$	Stagnation pressure ratio
7	$(4fL)/D$	Fanno line
8	$(s - s^*)/c_p$	Change in entropy

**3.6.2 arffam - Fanno relations given  $M$** 

```
void arffam(double g, double m, double result[8], int *ierr);
```

**Description**

Compute the fanno relations given a mach number,  $M$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M > 0$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 4.

**IERR - INTEGER EXIT:** (*optional*) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.

**3.6.3 arffap - Fanno relations given  $p/p^*$** 

```
void arffap(double g, double p, double result[8], int *ierr);
```

Compute the fanno relations given the pressure ratio,  $p/p^*$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**P - REAL ENTRY:** Pressure ratio  $0 \leq p/p^*$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 4.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $p/p^*$  out of range.

**3.6.4 arffad - Fanno relations given  $\rho/\rho^*$** 

```
void arffad(double g, double d, double result[8], int *ierr);
```

Compute the fanno relations given the density ratio,  $\rho/\rho^*$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**D - REAL ENTRY:** Density ratio  $\sqrt{(\gamma - 1)/(\gamma + 1)} \leq \rho/\rho^*$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 4.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $\rho/\rho^*$  out of range.

**3.6.5 arffat - Fanno relations given  $T/T^*$** 

```
void arffat(double g, double t, double result[8], int *ierr);
```

Compute the fanno relations given the temperature ratio,  $T/T^*$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**T - REAL ENTRY:** Temperature ratio  $0 \leq T/T^* \leq (\gamma + 1)/(\gamma - 1)$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 4.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $T/T^*$  out of range.



**3.6.6 arffav - Fanno relations given  $V/V^*$** 

```
void arffav(double g, double v, double result[8], int *ierr);
```

Compute the fanno relations given the velocity ratio,  $V/V^*$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**V - REAL ENTRY:** Velocity ratio  $0 \leq V/V^* \leq \sqrt{(\gamma+1)/(\gamma-1)}$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 4.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $V/V^*$  out of range.

**3.6.7 arffap0 - Fanno relations given  $p_{01}/p_{02}$** 

```
void arffap0(double g, double p0, bool is_sub, double result[8], int *ierr);
```

Compute the fanno relations given the stagnation pressure ratio,  $p_{01}/p_{02}$ .

**Performance**

Iterative

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**P0 - REAL ENTRY:** Stagnation pressure ratio  $1 \leq p_{02}/p_{01}$ .

**IS\_SUB - BOOLEAN ENTRY:** Whether  $M$  corresponding to the given  $p_{02}/p_{01}$  is assumed to be sub- or super- sonic.

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 4.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Computed  $M$  out of range.

**-3** Given  $p_{01}/p_{02}$  out of range.

**-4** Iterative solver failed.

**3.6.8 arffaf - Fanno relations given  $(4fL)/D$** 

```
void arffaf(double g, double l, bool is_sub, double result[8], int *ierr);
```

Compute the fanno relations given the fanno line,  $(4fL)/D$ .

**Performance**

Iterative

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**L - REAL ENTRY:** Fanno line  $0 \leq (4fL)/D \leq ((\gamma + 1)\log((\gamma + 1)/(\gamma - 1)) - 2)/(2\gamma)$ .

**IS\_SUB - BOOLEAN ENTRY:** Whether  $M$  corresponding to the given  $(4fL)/D$  is assumed to be sub- or super- sonic.

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 4.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $(4fL)/D$  out of range.
- 4 Iterative solver failed.

**3.6.9 arffads - Fanno relations given  $(s - s^*)/c_p$** 

```
void arffads(double g, double ds, bool is_sub, double result[8], int *ierr);
```

Compute the fanno relations given the fanno line,  $(s - s^*)/c_p$ .

**Performance**

Iterative

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**DS - REAL ENTRY:** Change in entropy  $-\infty < (s - s^*)/c_p < \infty$ .

**IS\_SUB - BOOLEAN ENTRY:** Whether  $M$  corresponding to the given  $(s - s^*)/c_p$  is assumed to be sub- or super- sonic.

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 4.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Computed  $M$  out of range.

**-3** Given  $(s - s^*)/c_p$  out of range.

**-4** Iterative solver failed.

## 3.7 Rayleigh Flow

### 3.7.1 Introduction

This function group computes the thermodynamic properties of non-adiabatic flow with heat addition for a calorically perfect gas as described in [JDAnderson1982].

The computed properties are as follows:

Table 5: Rayleigh flow properties

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M$	Mach number
2	$p/p^*$	Pressure ratio
3	$\rho/\rho^*$	Density ratio
4	$T/T^*$	Temperature ratio
5	$V/V^*$	Velocity ratio
6	$p_{01}/p_{02}$	Stagnation pressure ratio
7	$T_{01}/T_{02}$	Stagnation temperature ratio
8	$(s - s^*)/c_p$	Change in entropy

**3.7.2 arfram - Rayleigh relations given  $M$** 

```
void arfram(double g, double m, double result[8], int *ierr);
```

Compute the rayleigh relations given a mach number,  $M$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M > 0$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 5.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.

**3.7.3 arfrac - Rayleigh relations given  $p/p^*$** 

```
void arfrac(double g, double p, double result[8], int *ierr);
```

Compute the rayleigh relations given the pressure ratio,  $p/p^*$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**P - REAL ENTRY:** Pressure ratio  $0 < p/p^* < \gamma + 1$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 5.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $p/p^*$  out of range.

**3.7.4 arfrac - Rayleigh relations given  $\rho/\rho^*$** 

```
void arfrac(double g, double d, double result[8], int *ierr);
```

Compute the rayleigh relations given the density ratio,  $\rho/\rho^*$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

D - **REAL ENTRY**: Density ratio  $\gamma(\gamma + 1) < \rho/\rho^*$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 5.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $\rho/\rho^*$  out of range.



**3.7.5 arfrat - Rayleigh relations given  $T/T^*$** 

```
void arfrat(double g, double t, bool tmax, double result[8], int *ierr);
```

Compute the rayleigh relations given the temperature ratio,  $T/T^*$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**T - REAL ENTRY:** Temperature ratio  $0 < T/T^*$ .

**TMAX - BOOLEAN** Whether the static temperature ratio supplied is for a low speed false or a high speed true.

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 5.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $T/T^*$  out of range.

**3.7.6 arfrac - Rayleigh relations given  $V/V^*$** 

```
void arfrac(double g, double v, double result[8], int *ierr);
```

Compute the rayleigh relations given the velocity ratio,  $V/V^*$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**V - REAL ENTRY:** Velocity ratio  $0 < V/V^* < (\gamma + 1)/\gamma$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 5.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $V/V^*$  out of range.

**3.7.7 arfrap0 - Rayleigh relations given  $p_{02}/p_{01}$** 

```
void arfrap0(double g, double p0, bool is_sub, double result[8], int *ierr);
```

Compute the rayleigh relations given the stagnation pressure ratio,  $p_{02}/p_{01}$ .

**Performance**

Iterative

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**P0 - REAL ENTRY:** Stagnation pressure ratio  $p_{02}/p_{01} > \frac{\gamma^2-1}{\gamma^2}$ .

**IS\_SUB - BOOLEAN** Whether  $M$  corresponding to the given  $p_{02}/p_{01}$  is assumed to be sub- or super-sonic.

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 5.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $p_{02}/p_{01}$  out of range.
- 4 Iterative solver failed.

**3.7.8 arfrat0 - Rayleigh relations given  $T_{02}/T_{01}$** 

```
void arfrat0(double g, double t0, bool is_sub, double result[8], int *ierr);
```

Compute the rayleigh relations given the stagnation temperature ratio,  $T_{02}/T_{01}$ .

**Performance**

Iterative

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**T0 - REAL ENTRY:** Stagnation temperature ratio  $T_{02}/T_{01} \geq 0$ .

**IS\_SUB - BOOLEAN** Whether  $M$  corresponding to the given  $T_{02}/T_{01}$  is assumed to be sub- or super-sonic.

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 5.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $T_{02}/T_{01}$  out of range.
- 4 Iterative solver failed.

**3.7.9 arfrads - Rayleigh relations given  $(s - s^*)/c_p$** 

```
void arfrads(double g, double ds, bool is_sub, double result[8], int *ierr);
```

Compute the rayleigh relations given the fanno line,  $(s - s^*)/c_p$ .

**Performance**

Iterative

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**DS - REAL ENTRY:** Change in entropy  $-\infty < (s - s^*)/c_p < \infty$ .

**IS\_SUB - BOOLEAN** Whether  $M$  corresponding to the given  $(s - s^*)/c_p$  is assumed to be sub- or super- sonic.

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 5.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $(s - s^*)/c_p$  out of range.
- 4 Iterative solver failed.

## 3.8 Isothermal flow through long duct

### 3.8.1 Introduction

This function group computes an isothermal flow through a long ducts as described in [VLStreeter1966] for which neither fanno nor rayleigh flow is applicable due to the assumptions of the two latter.

The following properties are computed:

Table 6: Isothermal flow properties

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M$	Mach number
2	$\frac{p^*}{p}, \frac{r^*}{r}, \frac{M^*}{M}$	Non-stagnative ratios
3	$\rho/\rho^*$	Density ratio
5	$V/V^*$	Velocity ratio
6	$p_{01}/p_{02}$	Stagnation pressure ratio
6	$T_{01}/T_{02}$	Stagnation temperature ratio
7	$(4fL)/D$	Characteristic line

**3.8.2 arfitm - Isothermal relations given  $M$** 

```
void arfitm(double g, double m, double result[6], int *ierr);
```

Compute the isothermal relations given the mach number  $M$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M > 0$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 6.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.

**3.8.3 arfitv - Isothermal relations given  $V/V^*$** 

```
void arfitv(double g, double v, double result[6], int *ierr);
```

Compute the isothermal relations given the velocity ratio,  $V/V^*$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**V - REAL ENTRY:** Velocity ratio  $V/V^* > 0$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 6.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $V/V^*$  out of range.



**3.8.4 arfits - Isothermal relations given  $p^*/p, r^*/r, M^*/M$** 

```
void arfits(double g, double s, double result[6], int *ierr);
```

Compute the isothermal relations given a non-stagnative ratio,  $p^*/p, r^*/r, M^*/M$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

S - **REAL ENTRY**: Non-stagnative ratio  $p^*/p, r^*/r, M^*/M > 0$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 6.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given non-stagnative ratio out of range.

**3.8.5 arfitp0 - Isothermal relations given  $p_{01}/p_{02}$** 

```
void arfitp0(double g, double p0, bool is_sub, double result[6], int *ierr);
```

Compute the isothermal relations given the stagnation pressure ratio,  $p_{01}/p_{02}$ .

**Performance**

Iterative

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**P0 - REAL ENTRY:** Stagnation pressure ratio  $1 \leq p_{02}/p_{01}$ .

**IS\_SUB - BOOLEAN ENTRY:** Whether  $M$  corresponding to the given  $p_{02}/p_{01}$  is assumed to be sub- or super- sonic.

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 6.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Computed  $M$  out of range.

**-3** Given  $p_{01}/p_{02}$  out of range.

**-4** Iterative solver failed.

**3.8.6 arfitt0 - Isothermal relations given  $T_{02}/T_{01}$** 

```
void arfitt0(double g, double t0, double result[6], int *ierr);
```

Compute the isothermal relations given the stagnation temperature  $T_{02}/T_{01}$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

T0 - **REAL ENTRY**: Stagnation temperature  $T_{02}/T_{01} > 0$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 6.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

-1 Specific heat ratio  $\gamma \leq 1$ .

-2 Given  $T_{02}/T_{01}$  out of range.

**3.8.7 arfitf - Isothermal relations given  $(4fL)/D$** 

```
void arfitf(double g, double l, bool is_sub, double result[6], int *ierr);
```

Compute the isothermal relations given the characteristic line,  $(4fL)/D$ .

**Performance**

Iterative

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**F - REAL ENTRY:** Characteristic line  $(4fL)/D$ .

**IS\_SUB - BOOLEAN ENTRY:** Whether  $M$  corresponding to the given  $(4fL)/D$  is assumed to be sub- or super- sonic.

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 6.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $(4fL)/D$  out of range.
- 4 Iterative solver failed.

## 3.9 DeLaval nozzle flow

### 3.9.1 Introduction

This function group computes the properties of *quasi-2D* isentropic flow through a DeLaval nozzle, as described in [JDAnderson1982].

The following properties are computed:

Table 7: DeLaval nozzle properties

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M$	Mach number
2	$p/p_1$	Pressure ratio
3	$\rho/\rho^*$	Density ratio
4	$T/T_1$	Temperature ratio
5	$A/A_1$	Critical area ratio

**3.9.2 arfqism - DeLaval isentropic flow relations given  $M$** 

```
void arfqism(double g, double m, double result[5], int *ierr);
```

Compute the DeLaval nozzle isentropic relations given the mach number  $M$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M > 0$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 7.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.

**3.9.3 arfqisp - DeLaval isentropic flow relations given  $p/p_1$** 

```
void arfqisp(double g, double p, double result[5], int *ierr);
```

Compute the DeLaval nozzle isentropic relations given the pressure ratio  $p/p_1$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

P - **REAL ENTRY**: Pressure ratio  $0 < p/p_1 < 1$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 7.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $p/p_1$  out of range.

**3.9.4 arfqisd - DeLaval isentropic flow relations given  $\rho/\rho_1$** 

```
void arfqisd(double g, double d, double result[5], int *ierr);
```

Compute the DeLaval nozzle isentropic relations given the density ratio  $p/p_1$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

D - **REAL ENTRY**: Density ratio  $0 < \rho/\rho_1 < 1$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 7.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $\rho/\rho_1$  out of range.



**3.9.5 arfqist - DeLaval isentropic flow relations given  $T/T_1$** 

```
void arfqist(double g, double t, double result[5], int *ierr);
```

Compute the DeLaval nozzle isentropic relations given the temperature ratio  $T/T_1$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**T - REAL ENTRY:** Temperature ratio  $0 < T/T_1 < 1$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 7.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $T/T_1$  out of range.

**3.9.6 arfqisa - DeLaval isentropic flow relations given  $A/A^*$** 

```
void arfqisa(double g, double a, bool is_sub, double result[5], int *ierr);
```

Compute the DeLaval nozzle isentropic relations given the critical area ratio  $A/A^*$ .

**Performance**

Iterative

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**A - REAL ENTRY:** Critical area ratio  $A/A^* > 1$ .

**IS\_SUB - BOOLEAN ENTRY:** Whether the mach number corresponding to  $A/A^*$  is assumed to be sub- or super- sonic.

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 7.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $A/A^*$  out of range.
- 4 Iterative solver failed.

## 3.10 Normal Shock Waves

### 3.10.1 Introduction

These functions compute the thermodynamic properties across a normal shock wave, assuming a calorically perfect gas and an adiabatic and frictionless flow, as described in [naca1135]. It can compute the following set of properties given one of the properties in conjunction with a specific heat ratio  $\gamma$  of the gas medium.

The following equations govern normal shock waves as implemented in the function group:

Equation of Mass

$$p_1 u_1 = p_2 u_2$$

Equation of Momentum

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

Equation of Energy

$$\frac{1}{2} u_1^2 + h_1 = \frac{1}{2} u_2^2 + h_2 \text{ [adiab]}$$

Normal Shock Waves are classified by the following properties, where the subscript 1,2 refers to upstream and downstream, respectively.

The following properties are computed:

Table 8: Normal shock properties

I	Property	Description
1	$M_{1n}$	Upstream mach number
2	$M_{2n}$	Downstream mach number
3	$p_2/p_1$	Pressure ratio
4	$\rho_2/\rho_1$	Density ratio
5	$T_2/T_1$	Temperature ratio
6	$p_{02}/p_{01}$	Stagnation pressure ratio

**3.10.2 arsnsm1 - Normal shock relations given  $M_{1n}$** 

```
void arsnsm1(double g, double m, double result[5], int *ierr);
```

Compute the normal shock relations given the upstream mach number  $M_{1n}$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Upstream number  $M_{1n} \geq 1$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 8.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M_{1n}$  out of range.

**3.10.3 arsnsm2 - Normal shock relations given  $M_{2n}$** 

```
void arsnsm2(double g, double m, double result[5], int *ierr);
```

Compute the normal shock relations given the downstream mach number  $M_{2n}$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Upstream number  $M_{2n} > 0$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 8.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M_{2n}$  out of range.

**3.10.4 arsnsp - Normal shock relations given  $p_2/p_1$** 

```
void arsnsp(double g, double p, double result[5], int *ierr);
```

Compute the normal shock relations given the pressure ratio  $p_2/p_1$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

P - **REAL ENTRY**: Pressure ratio  $p_2/p_1 < 1$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 8.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M_{1n}$  out of range.
- 3 Given  $p_2/p_1$  out of range.

**3.10.5 arsnsd - Normal shock relations given  $\rho_2/\rho_1$** 

```
void arsnsd(double g, double d, double result[5], int *ierr);
```

Compute the normal shock relations given the density ratio  $\rho_2/\rho_1$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

D - **REAL ENTRY**: Density ratio  $1 \leq \rho_2/\rho_1 \leq (\gamma + 1)/(\gamma - 1)$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 8.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M_{1n}$  out of range.
- 3 Given  $\rho_2/\rho_1$  out of range.

**3.10.6 arsnst - Normal shock relations given  $T_2/T_1$** 

```
void arsnst(double g, double t, double result[5], int *ierr);
```

Compute the normal shock relations given the temperature ratio  $T_2/T_1$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

T - **REAL ENTRY**: Temperature ratio  $T_2/T_1 \geq 1$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 8.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M_{1n}$  out of range.
- 3 Given  $T_2/T_1$  out of range.



**3.10.7 arsnsp0 - Normal shock relations given  $p_{02}/p_{01}$** 

```
void arsnsp0(double g, double p0, double result[5], int *ierr);
```

Compute the normal shock relations given the stagnation pressure ratio  $p_{02}/p_{01}$ .

**Performance**

Iterative

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

P0 - **REAL ENTRY**: Stagnation pressure ratio  $0 \leq p_{02}/p_{01} \leq 1$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 8.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M_{1n}$  out of range.
- 3 Given  $p_{02}/p_{01}$  out of range.
- 4 Iterative solver failed.

## 3.11 Oblique Shock Relations

### 3.11.1 Introduction

This function group computes the thermodynamic properties of an oblique shock wave given either a mach number and the surface deflection or wave angle or based upon both the wave and deflection surface angles, using algorithms as described in [usafv1], [naca1135] and [NASA187173].

Oblique Shock Waves are classified by the following properties, where the subscript 1,2 refers to upstream and downstream, respectively. The normal mach numbers  $M_{1n}$  and  $M_{2n}$  represent the mach numbers through the shock front as by the normal shock wave equations.

The following properties will be available:

Table 9: Oblique shock properties

I	Property	Description
1	$M_1$	Upstream mach number
2	$M_2$	Downstream mach number
3	$M_{1n}$	Normal upstream mach number
4	$M_{2n}$	Normal downstream mach number
5	$\theta$	Deflection surface angle (°)
6	$\delta$	Wave angle (°)
7	$p_2/p_1$	Pressure ratio
8	$\rho_2/\rho_1$	Density ratio
9	$T_2/T_1$	Temperature ratio
10	$p_{02}/p_{01}$	Stagnation pressure ratio

**3.11.2 arsolmw - Oblique shock relations given  $M_1$  and  $\delta$** 

```
void arsolmw(double g, double m, double d, double result[10], int *ierr);
```

Compute the oblique shock relations given a mach number  $M$  and the wave angle  $\delta$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

M - **REAL ENTRY**: Mach number  $M_1 \geq 1$ .

D - **REAL ENTRY**: Wave angle  $0 < \delta < 90$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 9.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

-1 Specific heat ratio  $\gamma \leq 1$ .

-2 Given  $M_1$  out of range.

-3 Given  $\delta$  out of range.

**3.11.3 arsolpw - Oblique shock relations given  $p_2/p_1$  and  $\delta$** 

```
void arsolpw(double g, double p, double d, double result[10], int *ierr);
```

Compute the oblique shock relations given the pressure ratio  $p_2/p_1$  and the wave angle  $\delta$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

P - **REAL ENTRY**: Pressure ratio  $p_2/p_1 > -(\gamma - 1)/(\gamma + 1)$ .

D - **REAL ENTRY**: Wave angle  $0 < \delta < 90$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 9.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

-1 Specific heat ratio  $\gamma \leq 1$ .

-2 Given  $p_2/p_1$  out of range.

-3 Given  $\delta$  out of range.

**3.11.4** `arsoldw` - Oblique shock relations given  $\rho_2/\rho_1$  and  $\delta$ 

```
void arsoldw(double g, double ds, double d, double result[10], int *ierr);
```

Compute the oblique shock relations given the density ratio  $\rho_2/\rho_1$  and the wave angle  $\delta$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**DS - REAL ENTRY:** Density ratio  $0 < \rho_2/\rho_1 < (\gamma + 1)/(\gamma - 1)$ .

**D - REAL ENTRY:** Wave angle  $0 < \delta < 90$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 9.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Given  $\rho_2/\rho_1$  out of range.
- 3 Given  $\delta$  out of range.

**3.11.5** `arsoltw` - Oblique shock relations given  $T_2/T_1$  and  $\delta$ 

```
void arsoltw(double g, double t, double d, double result[10], int *ierr);
```

Compute the oblique shock relations given the temperature ratio  $T_2/T_1$  and the wave angle  $\delta$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

T - **REAL ENTRY**: Temperature ratio  $T_2/T_1$ .

D - **REAL ENTRY**: Wave angle  $0 < \delta < 90$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 9.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

-1 Specific heat ratio  $\gamma \leq 1$ .

-2 Given  $T_2/T_1$  out of range.

-3 Given  $\delta$  out of range.

**3.11.6** arsolp0w - Oblique shock relations given  $p_{02}/p_{01}$  and  $\delta$ 

```
void arsolp0w(double g, double p0, double d, double result[10], int *ierr);
```

Compute the oblique shock relations given the stagnation pressure  $p_{02}/p_{01}$  and the wave angle  $\delta$ .

**Performance**

Iterative

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

P0 - **REAL ENTRY**: Stagnation pressure ratio  $p_{02}/p_{01}$ .

D - **REAL ENTRY**: Wave angle  $0 < \delta < 90$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 9.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

-1 Specific heat ratio  $\gamma \leq 1$ .

-2 Given  $p_{02}/p_{01}$  out of range.

-3 Given  $\delta$  out of range.

-4 Iterative solver failed.

**3.11.7 arsolws - Oblique shock relations given  $\theta$  and  $\delta$** 

```
void arsolws(double g, double d, double t, double result[10], int *ierr);
```

Compute the oblique shock relations given the deflection surface angle  $\theta$  and the wave angle  $\delta$ .

**Performance**

Iterative

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

D - **REAL ENTRY**: Wave angle  $0 < \delta < 90$ .

T - **REAL ENTRY**: Deflection surface angle  $0 < \theta < 90$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 9.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

-1 Specific heat ratio  $\gamma \leq 1$ .

-3 Given  $\theta$  or  $\delta$  out of range.



**3.11.8 arsolms - Oblique shock relations given  $M_1$  and  $\theta$** 

```
void arsolms(double g, double m, double t, bool strong, double result[10], int *ierr);
```

Compute the oblique shock relations given the mach number  $M_1$  and the deflection surface angle  $\theta$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M_1 \geq 1$ .

**T - REAL ENTRY:** Deflection surface angle  $0 < \theta < 90$ .

**STRONG - BOOLEAN ENTRY:** Whether this is a strong shock.

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 9.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Given  $M_1$  out of range.
- 3 Given  $\theta$  out of range.
- 4 Calculation failed.

## 3.12 Oblique Shock Limits

### 3.12.1 Introduction

This set of functions computes the angle limits where an oblique shock would become detached or where the downstream flow would become sonic.

The following properties are computed:

Table 10: Oblique limit properties

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M$	Mach number for which the limit applies.
2	$\delta_{\max}$	Maximum wave angle ( $^{\circ}$ ).
3	$\theta_{\max}$	Maximum deflection surface angle ( $^{\circ}$ ).

**3.12.2** `arsolld` - Compute maximum wave and deflection surface angle before shock detachment

```
void arsolld(double g, double m, double result[3], int *ierr);
```

Compute maximum wave and deflection surface angle before shock detachment given a mach number  $M$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M \geq 1$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 10.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.

**3.12.3 arsolls - Compute maximum wave and deflection surface angle that wil result in sonic downstream flow**

```
void arsolls(double g, double m, double result[3], int *ierr);
```

Compute maximum wave and deflection surface angle that wil result in sonic downstream flow given a mach number  $M$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M \geq 1$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 10.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.

### 3.13 Prandtl-Meyer function

#### 3.13.1 Introduction

This function group contains a forward and inverse Prandtl-Meyer function, these two functions are separated from the expansion fan and normal shock function groups for convenience. The properties computed are as follows:

Table 11: Prandtl-Meyer properties

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M$	Mach number.
2	$v(M)$	Prandtl-Meyer function (°).
3	$\theta$	Mach angle (°).

**3.13.2 arspmm - Prandtl-Meyer properties given  $M$** 

```
void arspmm(double g, double m, double result[3], int *ierr);
```

Compute the Prandtl-Meyer properties given a mach number  $M$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M \geq 1$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 11.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.

**3.13.3 arspmv - Prandtl-Meyer properties given  $v(M)$** 

```
void arspmv(double g, double n, double result[3], int *ierr);
```

Compute the Prandtl-Meyer properties from the Prandtl-Meyer angle  $v(M)$ .

**Performance**

Iterative

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**N - REAL ENTRY:** Prandtl-Meyer angle  $v(M) \leq 90 \left( \sqrt{\frac{\gamma+1}{\gamma-1}} - 1 \right)$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 11.

**IERR - INTEGER EXIT:** (*optional*) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 3 Given  $v(M)$  out of range.
- 4 Iterative solver failed.

**3.13.4 arspmt - Prandtl-Meyer properties given  $\theta$** 

```
void arspmt(double g, double t, double result[3], int *ierr);
```

Compute the Prandtl-Meyer properties given a mach angle  $\theta$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

T - **REAL ENTRY**: Mach angle  $0 \leq \theta \leq 90$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 11.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $\theta$  out of range.



**3.13.5 arspmfm - Prandtl-Meyer angle from  $M$** 

```
double arspmfm(double g, double m);
```

Computes the Prandtl-Meyer angle  $v(M)$  from the given mach number  $M$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M \geq 1$ .

**Return value**

This function returns the Prandtl-Meyer angle  $v(M)$  for the given mach number  $M$  or NAN (Not-a-Number) if an error has occurred).

**3.13.6 arspmfv - Mach number from  $\nu(M)$** 

```
double arspmfv(double g, double v);
```

Computes the mach number  $M$  given the Prandtl-Meyer angle  $\nu(M)$ .

**Performance**

Iterative

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**V - REAL ENTRY:** Prandtl-Meyer angle  $\nu(M) \leq 90 \left( \sqrt{\frac{\gamma+1}{\gamma-1}} - 1 \right)$ .

**Return value**

This function returns the mach number  $M$  for the given mach Prandtl-Meyer angle  $\nu(M)$  or NAN (Not-a-Number) if an error has occurred).

### 3.14 Expansion Fan (Rarefaction Wave)

#### 3.14.1 Introduction

This class computes the thermodynamic properties of compressible gas flow over a wedge for a calorically perfect gas, as described by [naca1135]. It computes the following properties given the specific heat ratio  $\gamma$  and any of the up- or downstream mach number plus any of the other properties. After calling any of the evaluation functions the following properties will be available, where the subscript 1,2 refers to upstream and downstream, respectively;

The following properties are computed:

Table 12: Expansion fan properties

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M_1$	Upstream mach number.
2	$M_2$	Downstream mach number.
3	$p_2/p_1$	Pressure ratio.
4	$\rho_2/\rho_1$	Density ratio.
5	$T_2/T_1$	Temperature ratio.

**3.14.2 arsefm - Expansion fan properties given  $M_1$  and  $M_2$** 

```
void arsefm(double g, double m1, double m2, double result[5], int *ierr);
```

Compute the expansion fan properties given  $M_1$  and  $M_2$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

M1 - **REAL ENTRY**: Upstream mach number  $M_1 \geq 1$ .

M2 - **REAL ENTRY**: Downstream mach number  $M_2 > 0$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 12.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

-1 Specific heat ratio  $\gamma \leq 1$ .

-2 Given  $M_1$  or  $M_2$  out of range.

**3.14.3 arsefp - Expansion fan properties given  $p_2/p_1$  and either  $M_1$  or  $M_2$** 

```
void arsefp(double g, double p, double m, bool is_m2, double result[5], int *ierr);
```

Compute the expansion fan properties given  $p_2/p_1$  and either  $M_1$  or  $M_2$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**P - REAL ENTRY:** Pressure ratio  $p_2/p_1$ .

**M - REAL ENTRY:** Either  $M_1 \geq 1$  or  $M_2 > 0$  depending on whether `is_m2` is false or true, respectively.

**IS\_M2 - BOOLEAN ENTRY:** Whether the given `m` refers to  $M_1$  or  $M_2$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 12.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M_1$  or  $M_2$  out of range.

**-3** Given pressure ratio  $p_2/p_1$  out of range.

**3.14.4 arsefd - Expansion fan properties given  $\rho_2/\rho_1$  and either  $M_1$  or  $M_2$** 

```
void arsefd(double g, double d, double m, bool is_m2, double result[5], int *ierr);
```

Compute the expansion fan properties given  $\rho_2/\rho_1$  and either  $M_1$  or  $M_2$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**D - REAL ENTRY:** Density ratio  $\rho_2/\rho_1$ .

**M - REAL ENTRY:** Either  $M_1 \geq 1$  or  $M_2 > 0$  depending on whether `is_m2` is false or true, respectively.

**IS\_M2 - BOOLEAN ENTRY:** Whether the given `m` refers to  $M_1$  or  $M_2$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 12.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M_1$  or  $M_2$  out of range.

**-3** Given pressure ratio  $\rho_2/\rho_1$  out of range.

**3.14.5 arseft - Expansion fan properties given  $T_2/T_1$  and either  $M_1$  or  $M_2$** 

```
void arseft(double g, double t, double m, bool is_m2, double result[5], int *ierr);
```

Compute the expansion fan properties given  $T_2/T_1$  and either  $M_1$  or  $M_2$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**T - REAL ENTRY:** Temperature ratio  $T_2/T_1$ .

**M - REAL ENTRY:** Either  $M_1 \geq 1$  or  $M_2 > 0$  depending on whether `is_m2` is false or true, respectively.

**IS\_M2 - BOOLEAN ENTRY:** Whether the given `m` refers to  $M_1$  or  $M_2$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 12.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M_1$  or  $M_2$  out of range.

**-3** Given pressure ratio  $T_2/T_1$  out of range.

## 3.15 (Rayleigh-)Pitot Tube Relations

### 3.15.1 Introduction

Computes the mach number out of the corresponding pressure given or inverse. For the supersonic case it computes the Rayleigh-Pitot equation which takes the bow shock in front of the pitot tube into account.

The following properties are computed: For  $M < 1$  the Pitot pressure ratio  $p_0/p$  will be computed for  $M \geq 1$  the

Table 13: (Rayleigh-)Pitot properties

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M$	Mach number.
2	$p$	Pressure ratio, see below

Rayleigh-Pitot pressure ratio  $p_{02}/p_1$  will be computed.



**3.15.2 arsptm - (Rayleigh-)Pitot relations given  $M$** 

```
double arsptm(double g, double m, int *ierr);
```

Compute the Rayleigh-Pitot properties given a mach number  $M$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M > 0$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 13.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.

**Return Values**

The pressure ratio  $p_0/p$  or  $p_{02}/p_1$  or NAN on error.

**3.15.3 arsptp - (Rayleigh-)Pitot relations given  $p_0/p$  or  $p_{02}/p_1$** 

```
double arsptp(double g, double p0, bool is_sub, int *ierr);
```

Compute the Rayleigh-Pitot properties given a one of the pressure ratios  $p_0/p$  or  $p_{02}/p_1$ .

**Performance**

- Fixed if `is_sub` is true.
- Iterative if `is_sub` is false.

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**P0 - REAL ENTRY:** The pressure ratio  $p_0/p \geq 1$  or  $p_{02}/p_1 \geq ((1 + \gamma)/2)^{\gamma/(\gamma-1)}$  depending upon `is_sub`.

**IS\_SUB - BOOLEAN ENTRY:** Whether `p0` refers to a Pitot pressure fraction (true) or to a Rayleigh-Pitot pressure fraction (false).

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 13.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Given pressure ratio out of range.
- 3 Iterative solver failed.

**Return Values**

The Mach number  $M$  or NAN on error.

## 3.16 Reflected Shock Waves

### 3.16.1 Introduction

This function group computes the mach number of a reflected shock wave as given in [JDAnderson1982].

The following properties are computed:

Table 14: Reflected shock wave properties

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M$	Mach number of incident shock wave
2	$M'$	Mach number of reflected shock wave

**3.16.2 arsrms - Reflected shock wave mach number given  $M$** 

```
double arsrms(double g, double ms, int *ierr);
```

Computes the reflected shock wave mach number given  $M'$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**MS - REAL ENTRY:** Incident mach number  $M \geq 1$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 14.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.

**Return values**

The reflected shock wave mach number  $M'$  or NAN on error.

**3.16.3 arsrsmr - Incident shock wave mach number given  $M'$** 

```
double arsrsmr(double g, double mr, int *ierr);
```

Computes the incident shock wave mach number given  $M'$ .

**Performance**

Iterative

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M' > 0$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 14.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Given  $M'$  out of range.
- 3 Iterative solver failed.

**Return values**

The incident shock wave mach number  $M$  or NAN on error.

## 3.17 Quasi-2D Conical Flow

### 3.17.1 Introduction

This function group computes a solution to the *Taylor-Maccoll* equation for axis symmetric flow around a cone as for example described in [JDAnderson1982], that is:

$$\frac{\gamma-1}{2} \left[ 1 - V_r'^2 - \left( \frac{dV_r'}{d\theta} \right)^2 \right] \left[ 2V_r' + \frac{dV_r'}{d\theta} \cot\theta + \frac{d^2V_r'}{d\theta^2} \right] - \frac{V_r'}{d\theta} \left[ V_r' \frac{dV_r'}{d\theta} + \frac{dV_r'}{d\theta} \frac{d^2V_r'}{d\theta^2} \right] = 0$$

The solution to the above can be found given any combination of: the deflection surface angle  $\theta$  of the cone, an approximated wave angle  $\delta$  (i.e. using oblique shock relations) and the upstream mach number  $M$ . The solution is computed numerically using a Runge-Kutta method of the 4th order.

The following properties are computed:

Table 15: Conic shock properties

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M$	Mach number
2	$\theta$	Shock angle (°)
3	$\sigma$	Cone angle (°)

**3.17.2 arscoww - Conical flow given  $M$  and  $\delta$** 

```
void arscoww(double g, double m, double w, double result[3], int *ierr);
```

Solve the conical flow given a mach number  $M$  and wave angle  $\delta$ .

**Performance**

Iterative

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M \geq 1$ .

**W - REAL ENTRY:** Wave angle  $0 < \delta < 90$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 15.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Given  $M$  out of range.
- 3 Given  $\delta$  out of range.
- 4 Iterative solver failed.

**3.17.3 arscoms - Conical flow given  $M$  and  $\theta$** 

```
void arscoms(double g, double m, double s, double result[3], int *ierr);
```

Solve the conical flow given a mach number  $M$  and deflection surface angle  $\theta$ .

**Performance**

Iterative

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

M - **REAL ENTRY**: Mach number  $M \geq 1$ .

S - **REAL ENTRY**: Deflection surface angle  $0 < \theta < 90$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 15.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Given  $M$  out of range.
- 3 Given  $\theta$  out of range.
- 4 Iterative solver failed.



### 3.18 Moving Normal Shock Waves

Moving normal shock waves refer to normal shock waves in linear motion axis symmetric to the flow direction. Two sets of functions are available, one computes the thermodynamic properties of moving normal shock waves as described and the other set computes the mechanical/dynamic properties of such shock waves. These are separated as the functions computing the mechanical properties will require additional arguments in comparison to the thermodynamic property functions.

### 3.19 Moving Normal Shock Waves (Thermodynamic Properties)

#### 3.19.1 Introduction

This function group computes the thermodynamic properties of a moving normal shock wave.

The following properties are computed:

Table 16: Moving normal shock wave thermodynamic properties

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M$	Mach number
2	$p_2/p_1$	Pressure ratio
3	$\rho_2/\rho_1$	Density ratio
4	$T_2/T_1$	Temperature ratio

**3.19.2 arsmnsm - Moving shock relations given  $M$** 

```
void arsmnsm(double g, double m, double result[4], int *ierr);
```

Compute the moving normal shock wave relations given the mach number  $M$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $M > 0$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 16.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.

**3.19.3 arsmnsp - Moving shock relations given  $p_2/p_1$** 

```
void arsmnsp(double g, double p, double result[4], int *ierr);
```

Compute the moving normal shock wave relations given the pressure ratio  $p_2/p_1$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

P - **REAL ENTRY**: Pressure ratio  $p_2/p_1 > (1 - \gamma)/(\gamma + 1)$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 16.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $p_2/p_1$  out of range.

**3.19.4 arsmnsd - Moving shock relations given  $\rho_2/\rho_1$** 

```
void arsmnsd(double g, double d, double result[4], int *ierr);
```

Compute the moving normal shock wave relations given the density ratio  $\rho_2/\rho_1$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**D - REAL ENTRY:** Density ratio  $\rho_2/\rho_1 > -(\gamma^2 - 1)/(\gamma^2 + 1)$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 16.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $\rho_2/\rho_1$  out of range.

**3.19.5 arsmnst - Moving shock relations given  $T_2/T_1$** 

```
void arsmnst(double g, double t, double result[4], int *ierr);
```

Compute the moving normal shock wave relations given the temperature ratio  $T_2/T_1$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**T - REAL ENTRY:** Temperature ratio  $T_2/T_1 > (\gamma^2 + 1)/(\gamma + 1)^2$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 16.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Computed  $M$  out of range.
- 3 Given  $T_2/T_1$  out of range.

## 3.20 Moving Normal Shock Waves (Dynamic Properties)

### 3.20.1 Introduction

This function group computes the mechanic/dynamic properties of a moving normal shock wave. These not only require the specific heat constant  $\gamma$  but also the speed of sound in the latter medium  $a_0$ .

The following properties are computed:

Table 17: Moving normal shock waves dynamic properties

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$p_2/p_1$	Pressure ratio
2	$V$	Velocity
3	$U_P$	Max-motion velocity, downstream

**3.20.2 arsmnvp - Dynamic moving shock relations given  $p_2/p_1$** 

```
void arsmnvp(double g, double a0, double p, double result[3], int *ierr);
```

Compute the dynamic moving normal shock wave relations given the the pressure ratio  $p_2/p_1$  and the speed of sound  $a_0$  of the medium.

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

A0 - **REAL ENTRY**: Speed of sound in the medium  $a_0 > 0$ .

P - **REAL ENTRY**: Pressure ratio  $p_2/p_1 > (\gamma - 1)/(\gamma + 1)$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 17.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

- 1 Specific heat ratio  $\gamma \leq 1$ .
- 2 Given  $a_0$  out of range.
- 3 Given  $p_2/p_1$  out of range.

**3.20.3 arsmnvw - Dynamic moving shock relations given  $V$** 

```
void arsmnvw(double g, double a0, double w, double result[3], int *ierr);
```

Compute the dynamic moving normal shock wave relations given the the velocity  $V$  and the speed of sound  $a_0$  of the medium.

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**A0 - REAL ENTRY:** Speed of sound in the medium  $a_0 > 0$ .

**W - REAL ENTRY:** Velocity  $V > 0$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 17.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $a_0$  out of range.

**-3** Computed  $p_2/p_1$  out of range.

**-4** Given  $V$  out of range.



**3.20.4 arsmnvup - Dynamic moving shock relations given  $U_p$** 

```
void arsmnvup(double g, double a0, double up, double result[3], int *ierr);
```

Compute the dynamic moving normal shock wave relations given the mass-motion velocity  $U_p$  and the speed of sound  $a_0$  of the medium.

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**A0 - REAL ENTRY:** Speed of sound in the medium  $a_0 > 0$ .

**UP - REAL ENTRY:** Max-motion velocity  $U_p > -(2\sqrt{2}a_0\sqrt{\gamma(\gamma^2 + 2\gamma - 1)})/(\gamma + 1)$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 17.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $a_0$  out of range.

**-3** Computed  $p_2/p_1$  out of range.

**-4** Given  $U_p$  out of range.

### 3.21 Karman-Tsien Pressure Correction Coefficient

#### 3.21.1 Introduction

The *Karman-Tsien* pressure correlation coefficient is defined as:

$$C_p = \frac{C_{p0}}{\sqrt{1-M^2} + \frac{1}{2} \left( \frac{M^2}{1+\sqrt{1-M^2}} \right) C_{p0}}$$

Where  $C_{p0}$  the incompressibility coefficient.

The following properties are computed:

Table 18: Karman-Tsien property table

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M$	Mach number
2	$C_p$	Pressure correction coefficient
3	$C_{p0}$	Incompressibility coefficient

**3.21.2 arckarcp - Karman-Tsien pressure correction given  $M$  and  $C_{p0}$** 

```
void arckarcp(double g, double m, double cin, double result[3], int *ierr);
```

Compute the Karman-Tsien pressure correction coefficient relations given the mach number  $M$  and the the incompressibility coefficient  $C_{p0}$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

M - **REAL ENTRY**: Mach number  $0 < M < 0.8$ .

CP - **REAL ENTRY**: Incompressibility coefficient  $C_{p0}$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 18.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

-1 Specific heat ratio  $\gamma \leq 1$ .

-2 Given  $M$  out of range.

**3.21.3 arckarci - Karman-Tsien pressure correction given  $M$  and  $C_p$** 

```
void arckarci(double g, double m, double cp, double result[3], int *ierr);
```

Compute the Karman-Tsien pressure correction coefficient relations given the mach number  $M$  and the the Karman-Tsien pressure correction coefficient  $C_p$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $0 < M < 0.8$ .

**CP - REAL ENTRY:** Pressure correction coefficient  $C_p$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 18.

**IERR - INTEGER EXIT:** (*optional*) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.

## 3.22 Laitone Pressure Correction Coefficient

### 3.22.1 Introduction

The *Laitone* pressure correlation coefficient is defined as:

$$C_p = \frac{C_{p0}}{\sqrt{1 - M^2} + \left( M^2 \frac{1 + (\gamma - 1)/2 \times M^2}{2\sqrt{1 - M^2}} \right) C_{p0}}$$

Where  $C_{p0}$  the incompressibility coefficient.

The following properties are computed:

Table 19: Laitone property table

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M$	Mach number
2	$C_p$	Pressure correction coefficient
3	$C_{p0}$	Incompressibility coefficient

**3.22.2 arclaicp - Laitone pressure correction given  $M$  and  $C_{p0}$** 

```
void arclaicp(double g, double m, double cin, double result[3], int *ierr);
```

Compute the Laitone pressure correction coefficient relations given the mach number  $M$  and the the incompressibility coefficient  $C_{p0}$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $0 < M < 0.8$ .

**CP - REAL ENTRY:** Incompressibility coefficient  $C_{p0}$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 19.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.

**3.22.3 arclaici - Laitone pressure correction given  $M$  and  $C_p$** 

```
void arclaici(double g, double m, double cp, double result[3], int *ierr);
```

Compute the Laitone pressure correction coefficient relations given the mach number  $M$  and the the Laitone pressure correction coefficient  $C_p$ .

**Performance**

Fixed

**Arguments**

G - **REAL ENTRY**: Specific heat constant  $\gamma$ .

M - **REAL ENTRY**: Mach number  $0 < M < 0.8$ .

CP - **REAL ENTRY**: Pressure correction coefficient  $C_p$ .

RESULT - **ARRAY OF REAL EXIT**: Array with result properties as described in 19.

IERR - **INTEGER EXIT**: (optional) Return status code.

**Status codes**

-1 Specific heat ratio  $\gamma \leq 1$ .

-2 Given  $M$  out of range.

### 3.23 Prandtl-Glauert Pressure Correction Coefficient

#### 3.23.1 Introduction

The *Prandtl-Glauert* pressure correlation coefficient is defined as:

$$C_p = \frac{C_{p0}}{\sqrt{1-M^2}}$$

Where  $C_{p0}$  the incompressibility coefficient.

The following properties are computed:

Table 20: Prandtl-Glauert property table

<b>I</b>	<b>Property</b>	<b>Description</b>
1	$M$	Mach number
2	$C_p$	Pressure correction coefficient
3	$C_{p0}$	Incompressibility coefficient



**3.23.2 arcpglcp - Prandtl-Glauert pressure correction given  $M$  and  $C_{p0}$** 

```
void arcpglcp(double g, double m, double cin, double result[3], int *ierr);
```

Compute the Prandtl-Glauert pressure correction coefficient relations given the mach number  $M$  and the the incompressibility coefficient  $C_{p0}$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $0 < M < 0.8$ .

**CP - REAL ENTRY:** Incompressibility coefficient  $C_{p0}$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 20.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.

**3.23.3 arcpglci - Prandtl-Glauert pressure correction given  $M$  and  $C_p$** 

```
void arcpglci(double g, double m, double cp, double result[3], int *ierr);
```

Compute the Prandtl-Glauert pressure correction coefficient relations given the mach number  $M$  and the the Prandtl-Glauert pressure correction coefficient  $C_p$ .

**Performance**

Fixed

**Arguments**

**G - REAL ENTRY:** Specific heat constant  $\gamma$ .

**M - REAL ENTRY:** Mach number  $0 < M < 0.8$ .

**CP - REAL ENTRY:** Pressure correction coefficient  $C_p$ .

**RESULT - ARRAY OF REAL EXIT:** Array with result properties as described in 20.

**IERR - INTEGER EXIT:** (optional) Return status code.

**Status codes**

**-1** Specific heat ratio  $\gamma \leq 1$ .

**-2** Given  $M$  out of range.